Functional C++

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These words are my own.
This is slide code
Omitted

- noexcept propagation
Omitted

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- SFINAE-correctness
Omitted

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- nicer errors
Omitted

- noexcept propagation
- SFINAE-correctness
- nicer errors
- (almost) all of the metaprogramming.
Ommitted (cont)

- compile-time optimizations
Omitted (cont)

- compile-time optimizations
- lots of niceness
Omitted (cont)

- compile-time optimizations
- lots of niceness
- automatic inference
Omitted (cont)

- compile-time optimizations
- lots of niceness
- automatic inference
- and tons of other stuff.
C++ is not made for this. Yet. I have proposals in flight that will make this better.

- P2830 constexpr type sorting
- P2825 declcall
- P2826 Expression Aliases
- P2989 Universal Template Parameters
- P2841 Concept and variable-template template-parameters
The errors are bad.
The compile times are worse.
(but getting better)
The runtime performance is actually pretty great.
(something something alias analysis)
When in doubt, do the simple thing.
Functional approaches pay off in the large.
The Library

All the examples in this presentation actually run!

https://github.com/libfn/functional

By: Bronek Kozicki and Alex Kremer.

Theory: Bartosz Milewski and yours truly.

Under the ISC open-source license.
Optimize for correctness
Let the compiler prove our code.
Only plausible business logic should compile.
Tests are CRCs for our proofs.
The typesystem *is* a proof system.
Our brain does not fit a lot, and proofs are hard. We break down problems; 
but do we break down solutions?
Context is the greatest complectifier of proof.
(Thanks Tony & Hickey).
We usually prove code intuitively. It wouldn't work otherwise.
Tests are CRCs for our proofs. They detect some incorrectness. They don't prove code correct.
Context of a line of code

Explicit context:

• local variables
• function arguments
• your object has state
• ... and it changes!
Block context:

- loop condition (true in loop)
- loop postcondition (true after loop)
  - [inverse of loop condition!]
- current enclosing if-conditions
- negations of previous if-conditions (we're in the else block)
- catch (type)
- are we processing a sequence? (ranged for-loops)
Implicit context:

- global variables
- threadlocal variables
- syscall-accessible stuff
  - environment variables
  - filesystem
  - gid/uid
  - ...

Implicit context (cont):

- current exception (this is an implicit argument if you're in a Lippincott function!)
- which exceptions *may* be thrown by subcalls
- cursors of open files
- open sockets / connections
- shared memory regions
Implicit context (cont 2):

- dispatched async operations waiting to complete
  - how do I cancel the current operation? Is it even cancellable?
- operating system limits
- global subsystems (Singletons)
  - logging
  - execution resources
  - memory allocator
  - shared network connections
Implicit context (cont 3):

- state machines: which *state* you're in?
- What execution context is running the code?
  - high priority thread
  - main thread
  - low priority thread
  - top-half of a signal handler
  - GPU, NUMA node, IO thread, compute thread, which threadpool?????
For every Context grows a Monad.

-- Teta Pehta

("For every disease there grows a flower")
Monad is meant loosely - I mean a compositional context.

Composition is the key.

Functor, Applicative, Monad, and some other things are all valid here.

We also need indexed monads and graded monads.
Limiting context

Most context is irrelevant at any one time. *Which* part is up to convention and discipline. Procedural languages have problems encoding proofs about context.
In functional languages, all context is explicit.
This helps proofs.
Can we be disciplined in C++?
Tools

We will need tools.
Banish all context-accessors from business logic into named contexts.
And we will explicitly give access.
And we will be free.
Apprentice: But master, my function has 43 parameters?

Master: you were blind, but now you see.
Design for laziness: Make wrong things painful. This is good. It allows more of the program to be context-free.

Push control flow, mutation, and I/O towards main().

Also see Tony's "Complecting made Easy".
Zen of python: flat is better than nested. Have you considered flattening out your dependencies?
Classic monads

- Maybe: optional
- Error: expected
- List: `std::ranges`
- Async: `std::execution`
- I/O: `std::execution IO-resources`
- State: what do you think OOP without PLOP is?
- Environment: `std::execution` has `read()` and `write()`
The final guideline

Name common patterns and control flow.
optional - the Maybe monad
Pattern 1: short-circuit

Bind:

\[ \text{and\_then :: } (T \to \text{opt}<U>) \to (\text{opt}<T> \to \text{opt}<U>) \]

Pattern:

\[ \text{opt ? f(*opt) : nullopt;} \]

Example:

```
auto parse_url(std::string) -> optional<URL>;

auto result
    = jsonfile.get_string("url") // optional<string>
    | and_then(parse_url)       // optional<URL>
    | and_then(http_get);       // optional<HTTP_RESULT>
```
Pattern 2: defaulting

Bind:

\[
\text{or\_else :: } ((\cdot) \rightarrow \text{opt}\langle T \rangle) \rightarrow (\text{opt}\langle T \rangle \rightarrow \text{opt}\langle T \rangle)
\]

Pattern: *Try multiple things and return the first one that succeeded.*

\[
\text{opt} ? \text{opt} : f();
\]

Extended example follows.
Procedural way of implementing get_hostname:

```cpp
auto is_hostname_in_args(int, char const* const*) -> bool;
auto get_hostname_from_args(int, char const* const*) -> char const*;

auto get_hostname(int argc, char const* const* argv,
                  std::string default_hostname)
    -> std::string {
    // split query / getter
    if (is_hostname_in_args(argc, argv)) {
        // perhaps... might use optional here too?
        return get_hostname_from_args(argc, argv);
    }
    // ad-hoc Maybe
    if (char const* maybe_host = getenv("SERVICE_HOSTNAME");
        (maybe_host != nullptr) && (*maybe_host != '\0')) {
        return maybe_host;
    }
    return default_hostname;
}
```
Functional rewrite

First, adapt to uniformity.
auto is_hostname_in_args(int, char const* const*) -> bool;
auto get_hostname_from_args(int, char const* const*) -> char const*;

constexpr auto maybe_hostname_from_args = [](int argc, char const* const* argv) -> std::optional<std::string>
{
    if (is_hostname_in_args(argc, argv)) {
        return get_hostname_from_args(argc, argv);
    }
    return std::nullopt;
};
From: nullptr-on-nothing

```cpp
auto std::getenv(char const*) -> char const*;
```

To: nullopt-on-nothing

```cpp
constexpr auto get_env = [](std::string const& varname) -> optional<std::string>
{
    if (char const* value = std::getenv(varname.c_str());
        value != nullptr) {
        return optional<std::string>(value);
    }
    return {std::nullopt};
};
```
Tool: filter

For the "nonempty" bit, we'll want a filter:

```cpp
constexpr auto filter = [](auto predicate) {
    return [=]<class T>(T&& value) -> optional<std::remove_cvref_t<T>>{
        if (predicate(value)) {
            return value;
        }
        return std::nullopt;
    };
};
```
Tool: filter

For the "nonempty" bit, we'll want a filter:

```cpp
constexpr auto filter = [](auto predicate) {
    return [=]<class T>(T&& value) -> optional<std::remove_cvref_t<T>> {
        if (predicate(value)) {
            return value;
        }
        return std::nullopt;
    };
};
```
First pass: put it together:

```cpp
constexpr auto nonempty = [] (auto const & s) { return !s.empty(); };

auto get_hostname(
    int argc, char const * const * argv,
    std::string const & default_hostname)
-> std::string {
    return maybe_hostname_from_args(argc, argv)
    | or_else([]{
        return get_env("SERVICE_HOSTNAME")
        .and_then(filter(nonempty));
    })
    // can add other ways
    | value_or(auto(default_hostname));
}

Blech.
```
First pass: put it together:

```cpp
constexpr auto nonempty = [](auto const& s){return !s.empty();};

auto get_hostname(int argc, char const* const* argv,
std::string const& default_hostname) -> std::string {
    return maybe_hostname_from_args(argc, argv)
        | or_else([]{
            return get_env("SERVICE_HOSTNAME")
                .and_then(filter(nonempty));
        })
        // can add other ways
        | value_or(auto(default_hostname));
}
```

Blech.
We defined filter wrong. Let's fix it.

```cpp
1 constexpr struct filter_t {
2   template <typename P> struct closure { P pred; };
3
4   template <typename P>
5   constexpr auto operator() (P&& predicate) const {
6     return closure{std::forward<P>(predicate)};
7   }
8
9   template <fn::some_optional Opt, typename P>
10  friend constexpr auto operator|(Opt&& opt, closure<P> const& cl) {
11     return std::forward<Opt>(opt).and_then(
12       [&] (auto&& v) -> std::remove_cvref_t<Opt> {
13         if (cl.pred(v)) {
14           return v;
15         }
16       return std::nullopt;
17     });
18   } filter;
```
We defined filter wrong. Let's fix it.

```cpp
constexpr struct filter_t {
    template <typename P> struct closure { P pred; }; 

template <typename P>
constexpr auto operator() (P&& predicate) const {
    return closure{std::forward<P>(predicate)}; 
}

template <fn::some_optional Opt, typename P>
friend constexpr auto operator|(Opt&& opt, closure<P> const & cl) {
    return std::forward<Opt>(opt).and_then(
        [&](auto&& v) -> std::remove_cvref_t<Opt> {
            if (cl.pred(v)) {
                return v;
            } else {
                return std::nullopt;
            }
        });
}
}
```
We defined filter wrong. Let's fix it.

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constexpr struct filter_t {
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}

template <fn::some_optional Opt, typename P>
friend constexpr auto operator |(Opt&& opt, closure<P> const & cl) {
    return std::forward<Opt>(opt).and_then([&](auto && v) -> std::remove_cvref_t<Opt> {
        if (cl.pred(v)) {
            return v;
        }
        return std::nullopt;
    });
}

} filter;
```
We defined filter wrong. Let's fix it.

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}

template <fn::some_optional Opt, typename P>
friend constexpr auto operator|(Opt&& opt, closure<P> const& cl) {
    return std::forward<Opt>(opt).and_then([&](auto&& v) -> std::remove_cvref_t<Opt> {
        if (cl.pred(v)) {
            return v;
        }
        return std::nullopt;
    });
}
}
```
We can omit the `and_then`:

```cpp
auto get_hostname(
    int argc, char const* argv,
    std::string const& default_hostname)
    -> std::string {
    return maybe_hostname_from_args(argc, argv)
    | or_else([]{ return get_env("SERVICE_HOSTNAME")
                 | filter(nonempty); })
    | value_or(auto(default_hostname));
}
```
Aside: some_optional

We needed a concept for the above:

```cpp
// fastest-to-compile is_specialization_of I know of
template <typename X>
constexpr bool _some_optional_v = false;

template <typename T>
constexpr bool _some_optional_v<fn::optional<T>&> = true;

template <typename T>
constexpr bool _some_optional_v<fn::optional<T> const&> = true;
// same for std::optional

template <typename X>
concept some_optional = _some_optional_v<X&>;
```

fn::some_optional of course already exists; but you will need this for your own types.
Everything called some_{XXX} is a concept like the above.
Fix the defaulting.

If you don't need to make a choice, why make it?

Just let the caller do it:

```cpp
auto maybe_hostname_from_env(int argc, char const* const* argv)
  -> std::optional<std::string> {
    return maybe_hostname_from_args(argc, argv)
       .or_else([]{
            return get_env("SERVICE_HOSTNAME")
               | filter(nonempty); });

  // <-- no more default
```
This isn't always better - you might choose to do it in one or the other location depending on the desired semantics.
Recap: function "lifts" on optional<T>:

- `and_then` :: (T -> opt<U>) -> (opt<T> -> opt<U>);
- `transform` :: (T -> U) -> (opt<T> -> opt<U>);
- `or_else` :: (() -> opt<T>) -> (opt<T> -> opt<T>);
- `value_or` :: T -> (opt<T> -> T);

`value_or` is an escape operation - final default.
expected: A monad for errors

Instead of nullopt, we get E.

The flavor is different. Error is error, maybe is maybe.
There are as many expecteds as there are Es.
Function lifts on expected\langle T, \ E\rangle

- \++ \ transform\_error
- \ or\_else now takes the error type.
- \ and\_then cannot change \ E.
- Neither can \ or\_else.

This means we actually got an optional-per-\ E.
expected is a domain-based composition type. It is *not* better exceptions.
Pattern: short-circuit

<table>
<thead>
<tr>
<th>Function</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>and_then</code></td>
<td><code>(T -&gt; exp&lt;U, E&gt;) -&gt; (exp&lt;T, E&gt; -&gt; exp&lt;U, E&gt;)</code></td>
</tr>
<tr>
<td><code>transform</code></td>
<td><code>(T -&gt; U) -&gt; (exp&lt;T, E&gt; -&gt; exp&lt;U, E&gt;)</code></td>
</tr>
<tr>
<td><code>or_else</code></td>
<td><code>(E -&gt; exp&lt;T, E&gt;) -&gt; (exp&lt;T, E&gt; -&gt; exp&lt;T, E&gt;)</code></td>
</tr>
<tr>
<td><code>transform_error</code></td>
<td><code>(E -&gt; E') -&gt; (exp&lt;T, E&gt; -&gt; exp&lt;T, E&gt;')</code></td>
</tr>
<tr>
<td><code>value_or</code></td>
<td><code>T -&gt; (exp&lt;T, E&gt; -&gt; T)</code></td>
</tr>
</tbody>
</table>

Happy monad:

- `and_then` lifts functions that can "fail".
- `transform` is a "bridge" - lifts functions that can't fail.

Sad monad:

- `or_else` is "recover" - the other bind.
- `transform_error` is the error bridge.
Parsers: a good fit for expected

Compositional parser infrastructure.

Every parser's signature is

```cpp
struct ParseError { std::span<char const> where; };
auto subgrammar(std::span<char const>)
    -> std::expected<pair<SomeGrammarNode, std::span<char const>>, ParseError>;
```
For instance, parse_int

template <typename T>
using parser_for = fn::expected<std::pair<T, std::span<char const>>, ParseError>, ParseError
auto parse_int(std::span<char const> src) {
    --> parser_for<int> {
        if (src.empty()) { return unexpected(ParseError(src)); } 
        bool negative = src[0] == '-';
        if (negative || src[0] == '+') { src = src.subspan(1); }
        if (src.empty()) { return unexpected(ParseError(src)); } 
        int result = 0;
        if ('0' <= src[0] && src[0] <= '9') {
            result = src[0] - '0';
            src = src.subspan(1);
        }
        while (!src.empty() && '0' <= src[0] && src[0] <= '9') {
            // not checking for overflow
            result = result * 10 + (src[0] - '0');
            src = src.subspan(1);
        }
        return std::pair{result * (-1 * negative), src};
    }
}
We could rework this to reduce the number of visible branches:

```cpp
auto parse_digit(std::span<char const> src) -> parser_for<int> {
    return (!src.empty() && '0' <= src[0] && src[0] <= '9')
        ? parser_for<int>(std::pair{src[0] - '0', src.subspan(1)});
        : unexpected(ParseError(src));
}

auto parse_positive_digit(std::span<char const> src) -> parser_for<int>
    return parse_digit(src).and_then([&](auto r) {
        return r.first > 0 ? parser_for<int>(r)
            : unexpected(ParseError(src));
    });
```
Rework the "integral" portion...

```cpp
auto parse_unprefixed_int(std::span<char const> src)
    -> fn::expected<std::pair<int, std::span<char const>>, ParseError> {
    return parse_positive_digit(src)
        .transform([&](auto p){
            auto&& [result, rest] = p;
            while (auto d = parse_digit(rest)) {
                // not checking for overflow
                result = result * 10 + d.value().first;
                rest = d.value().second;
            }
            return std::pair{result, rest};
        });
}
```
auto parse_maybe_sign(std::span<char const> src) -> parser_for<int> {
    if (src.empty()) { return unexpected(ParseError(src)); }
    bool negative = src[0] == '-';
    if (negative || src[0] == '+') {
        return std::pair{negative: -1:1, src.subspan(1)};
    }
    return parse_digit(src) // just check it's a digit
        .transform([&](auto&&) { return std::pair{1, src}; });
}
And finally put it all together:

```cpp
auto parse_int(std::span<char const> src) -> parser_for<int> {
    return parse_maybe_sign(src)
        .and_then([](auto sign) {
            return parse_unprefixed_int(sign.second)
                .transform([&](auto p){
                    return std::pair{sign.first * p.first, p.second};
                });
        });
}
```
Seems limited.

It is.

Problem: `std::expected` is not exceptions. We can't also return `IOError`. 
Towards multiple errors

Given:

template <typename T, typename... Ts>
concept any_of =
 (... || std::same_as<std::remove_cvref_t<T>, Ts>);
C++ has exceptions, and we want to catch any of them:

```cpp
try {
  g(2); // throws... what?
} catch (any_of<E1, E2, E3> auto && exc) {
  /* no you don't says C++ */
} // did I get all of them?
```

You need to know what `g` throws.

- Need polymorphic runtime matching.
- Need generic matching.
C++ has exceptions, and we want to catch any of them:

```
try {
  g(2); // throws... what?
} catch (any_of<E1, E2, E3> auto&& exc) {
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} // did I get all of them?
```

You need to know what `g` throws.

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You need to know what `g` throws.

- Need polymorphic runtime matching.
- Need generic matching.
How about:

```cpp
// auto g(int i) -> expected<int, variant<E1, E2, E3>>;
auto r = g(2).transform_error([](auto&& v) {
    return std::visit([](any_of<E1, E2, E3> auto&&) {
        return E2{};
    });
});
// r :: expected<int, E2>
```

Alright-ish, but ...
It doesn't compose.

variant is not useful for errors. visit does not merge types.

Variant is an index-discriminated union. We need a union.
sum<Ts...>

- Ts... are sorted and uniqued.

```
transform :: {(Ti -> sum<Uij...>) ; i,j} -> (sum<Ti...> -> sum<Uij...>; i,j)
```

1. given an overload set that can handle any Ti in Ts...
2. wrap every return type in sum (and flatten)
3. join them; that's the return type.
4. Apply the overload set to the active member
5. embed into actual return type.
join for sums

\[
\begin{align*}
\text{join<sum<Ts...>}, \text{sum<Us...>>} & \equiv \text{sum<Ts..., Us...>} \\
\text{join<sum<Ts...>}, \text{T} & \equiv \text{sum<Ts..., T>} \\
\text{join<T, sum<Ts...>>} & \equiv \text{sum<T, Ts...>}
\end{align*}
\]

join sorts and uniques the types for sums.
`sum_for<int, string>{2}
 | transform(overload{
     [](int x) -> sum_for<NegativeInt, PositiveInt> {
         if (x < 0) {
             return NegativeInt{x};
         }
         return PositiveInt{x};
     },
     fn::identity
 }); // sum_for<string, NegativeInt, PositiveInt>`
We also define it for optional
It has to look \textit{inside}:

\begin{verbatim}
join<
    optional<T>,
    optional<U>,
> == optional<join<T, U>>
\end{verbatim}

\texttt{join} is not defined for arbitrary types!
All monadic operations need to "obey" what `transform` means for `sum`.

```cpp
optional<sum<string, >>{}
| or_else([]{{ return optional{42}; }})
// optional<sum_for<int, string>>
```
what about `expected`?

Has to look inside on both sides.

```java
join<
  expected<T, E1>,
  expected<U, E2>,
> == expected<join<T, U>, join<E1, E2>>
```
`parse_line(string) -> expected<Result, ParseError>;

read_line(file) // expected<string, IOError>
 | sum_error // expected<string, sum<IOError>>
 | and_then(parse_line) // expected<Result, sum<IOError, ParseError>>
All other monads follow the same pattern.
Back to exceptions

Just works!

```cpp
// auto g(int i) -> expected<int, sum<E1, E2, E3>>;
g(2) | transform_error(overload{
    [](any_of<E1, E3> auto&&) {
        return E1{};
    },
    [](E2 x) {print(x); return x;}
});
```
You don't handle every exception? Fail to compile.

```
// auto g(int i) -> expected<int, sum<E1, E2, E3>>;
g(2) | transform_error(
    [](any_of<E1, E3> auto&&) {
        return E1{};
    }
);
// ERROR: not callable with E2
```
You want to let some through? Do it explicitly:

```cpp
// auto g(int i) -> expected<int, sum<E1, E2, E3>>;
auto r = g(2).transform_error(
    fn::overload{
        [](E1 const&) { return E2{}; },
        fn::identity /* id-on-E2 and E3 */
    }
); // expected<int, sum<E2, E3>>
```
Packs stand for "multiple arguments".

They have their own operation - `join`

```cpp
pack{1, "asdf"s}
| transform([](int x, string y) { return pack{x, y.size(), x+y};
| transform([](int x, size_t y, size_t z) { std::cout <<x << y <<
```
cat for packs

pack<Ts...> & pack<Us...> == pack<Ts..., Us...>
pack<Ts...> & U == pack<Ts..., U>
U & pack<Ts...> == pack<U, Ts...>
pack<pack<Ts...>> == pack<Ts...>

Packs autoflatten and concatenate unpacked arguments.
It still distributes inside monads, same as `join`. Multiple monadic arguments work as packs!

```cpp
struct config {
    template <typename T>
    auto get_maybe(std::string const & key) -> fn::optional<T>;
};

struct socket_addr { std::string hostname; int port; };

auto connect(config c) -> fn::optional<socket_addr> {
    return (c.get_maybe<std::string>("hostname")
            & c.get_maybe<int>("port"))
        // optional<pack<std::string, int>>
        | transform([](std::string && hostname, int port){
            return socket_addr{std::move(hostname), port};
        });
}
```
Creating objects is common.

We name common patterns.

```cpp
template <typename T>
constexpr auto make = []<typename... Ts>(Ts&&... args) {
    return T(std::forward<Ts>(args)...);
};

auto connect(config c) -> std::optional<socket_addr> {
    return (c.get_maybe("hostname")
        & c.get_maybe("port"))
        | transform(make<socket_addr>);
    // ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
}
```
auto parse_response(json const& doc) -> fn::expected<Response, ParseError> {
    return (doc.get_maybe("version")
            | or_else([]){return ParseError("version is required");})
            | parse_version
            | filter(eq(version{3, 14}), make<ParseError>)
    ) // expected<Version, ParseError>
    & (doc.get_maybe("id")
        | or_else([]){return ParseError("id is required");})
        | parse_int
        | transform(make<Id>)
    ) // expected<Id, ParseError>
    & /*...*/
    ) // expected-pack<Version, Id, ..., ParseError>
    | transform(make<Response>);
• We didn't forget error checking.
• No way to make a Response on failure.
• We validate everything due to strong types.
Strong types

- Version is just an integer
- constructor validates
- Response takes a Version
  - We can't make a Response without a Version
Only plausible business logic should compile.
Encode things in types until that is true.
Price is a point-space over the integral module of PriceDelta.

std::chrono does this too.
Making returning and calling symmetric

sum is special:
transform finds the common return type (it's a sum)

```c++
sum<A, B, C>{b} | transform(overload{
    [](B) {...} -> sum<E, F>,
    [](A) {...} -> sum<pack<C, D>, E>,
    [](C) {...} -> G,
}); // sum<E, F, G, pack<C, D>>
```
pack unpacks into arguments

pack{1, A{}, B{}} | transform([](int, A, B) {});
pack of sums does multiple dispatch

pack<sum<A, B>, sum<X, Y>>{A{}, B{}}
| transform{overload{
  [](A, X) {...},
  [](A, Y) {...},
  [](B, X) {...},
  [](B, Y) {...}
}}:

... and it also finds the common type.
The algebra

For ONE T:

\[ \text{sum}<T> = \text{pack}<T> = \text{sum}<\text{pack}<T>> = \text{pack}<\text{sum}<T>> \]
For multiple:

\[
\text{pack}<\text{sum}<A, B>, \text{sum}<X, Y>> = \text{sum}<\text{pack}<A, X>, \text{pack}<A, Y>, \text{pack}<B, X>, \text{pack}<B, Y>>
\]

(works for more than one sum in a pack)
Flattening

$\text{sum} \langle \text{sum} \langle T, U \rangle \rangle \rightarrow \text{sum} \langle T, U \rangle$

$\text{pack} \langle \text{pack} \langle T, U \rangle, V \rangle \rightarrow \text{pack} \langle T, U, V \rangle$.

These equivalences let us define calling overload sets.
An overload set is a function defined "in parts". Overload set calls don't compose; you can call it once, but you can't call one with a return value (you only get one type).

This makes pipelines not work.
A state machine:

```cpp
using State = sum<Initialized, Connecting, Connected, Disconnected, Fail>;
using Message = sum<Connect, Stop, Data, UnexpectedDisconnect>;
template <typename X>
concept some_live_state = any_of<X, Initialized, Connecting, Connected>;

auto transition(State s, Message m) {
    return (s & m) // pack<State, Message>
        | transform(fn::overload{ // double dispatch!
            [](Initialized s, Connect c)     -> sum<Connecting, Fail> {...},
            [](Connected s, Data d)          -> sum<Connected, Fail>  {...},
            [](some_live_state auto s, Stop) -> Disconnected          {...},
            [](some_live_state auto s, UnexpectedDisconnect)
                -> sum<Connecting, Fail> {...},
            [](auto s, auto m)               -> Fail                  {...}
        }); // -> <Connecting, Fail, Connected, Disconnected>
}
```
Enter graded monads

Graded monads are... relaxed.

```cpp
auto version_or_error = open(path) // fn::expected<File, sum<DoesNotExist, PermError|
| and_then(read_line // expected<std::string, IOError>
) // expected<std::string, sum<DoesNotExist, PermError, IOError, SyntaxError>
| and_then(parse_version);
// expected<Version, sum<DoesNotExist, PermError, IOError, SyntaxError>
```
expected<T, sum<...>> is a closed-polymorphic equivalent to c++ exceptions.

We can also handle the errors generically.

```cpp
version_or_error
 | on_error( // may return void
  overload{
    [](any_of<DoesNotExist, PermError, IOError> auto&& e) {
      std::print("could not read file {}", e.filename);
    },
    [](SyntaxError const& e) {
      std::print("syntax error near column {}", e.col);
    }
  });
```
Back to `get_maybe`...

Using `sum`, drop the type arg of `get_maybe`:

```cpp
struct config {
    auto get_maybe(std::string const& key)
        -> fn::optional<fn::sum_for<std::string, int, float>>;
};

auto connect(config c) -> fn::optional<socket_addr> {
    return (c.get_maybe("hostname") & c.get_maybe("port"))
         //              ^ <std::string> no longer here
         // optional<pack<sum<...>, sum<...>>, sum<...>>
         | and_then(fn::overload{
             [](std::string && hostname, int port) {
                 return fn::optional(socket_addr{std::move(hostname), port});
             },
             // fail if types weren't correct
             [](auto const&...) { return std::nullopt; }
         });
```
Drilling

What if you have a monad in a monad?

```csharp
expected<optional<int>, E>{optional{3}}
    | transform(
    transform(
        [](int x){ return format("{}", x); }
    )
); // expected<optional<string>>
```
Congratulations on surviving this gentle introduction to the sum-pack kaboodle

Making overload sets composable since 2023.
Conclusion

In order to name common control flow patterns, we reach for named compositional contexts, some of which are monads.

We explored the following contexts today:

- optional or "Maybe"
- expected or "Error"
- sum-pack kaboodle

And we touched on what it takes to make function call composition work the same way going out as going in.
Happy hacking!
Diagrams:

- regular monad-in-a-2-category diagram
- graded monad-in-a-2-category diagram